Name..ID............................Section...............Seat No..............

Sirindhorn International Institute of Technology
Thammasat University

Midterm Examination: Semester 1 / 2016

Course Title: ECS332 (Principles of Communications)

Instructor: Asst. Prof. Dr.Prapun Suksompong

Date/Time: October 12, 2016 / 13:30 - 16:30

Instructions:

- This examination has ....9.... pages (including this cover page).
- Conditions of Examination:

  ....Closed book

  (No dictionary, ☐ No calculator ☑ Calculator (e.g. FX-991) allowed)

  ....Open book

  .....✓....**Semi-Closed book** (.......1........sheet(s) ☑ 1 page ☐ both sides of A4 paper note)

  This sheet must be hand-written.
  Do not modify (,e.g., add/underline/highlight) content on the sheet inside the exam room.
  It should be **<u>submitted with the exam</u>**.
  Other requirements are specified on the course web site. (-10 pt if not following the requirements.)
- **<u>Read</u> these instructions and the questions carefully**.
- Students are not allowed to be out of the examination room during examination. Going to the restroom may result in score deduction.
- Turn off all communication devices and place them with other personal belongings in the area designated by the proctors or outside the test room.
- Write your name, student ID, section, and seat number clearly in the spaces provided on the top of this sheet. Then, write your **first name and the <u>last three digits</u> of your ID** in the spaces provided on the top of each page of your examination paper, starting from page 2.
- <u>The back of each page</u> will <u>not</u> be graded; it can be used for calculations of problems that do not require explanation.
- The examination paper is not allowed to be taken out of the examination room. Also, do not remove the staple. Violation may result in score deduction.
- Unless instructed otherwise, ***<u>write down all the steps</u>*** that you have done to obtain your answers.
  - When applying formula(s), state clearly which formula(s) you are applying before plugging-in numerical values.
  - You may not get any credit even when your final answer is correct without showing how you get your answer.
  - Formula(s) not discussed in class can be used. However, derivation must also be provided.
  - Exception: The 1-pt parts are graded solely on your answers. For these parts, there is no partial credit and it is not necessary to write down your explanation.
- When not explicitly stated/defined, all notations and definitions follow ones given in lecture. For example, the sinc function is defined by $\text{sinc}(x) = (\sin x)/x$; time is denoted by t and frequency is denoted by f. The unit of t is in seconds and the unit of f is in Hz.
- Some points are reserved for *accuracy* of the answers and also for reducing answers into their *simplest* forms. Watch out for roundoff error.
- Points marked with * indicate challenging problems.
- Do not cheat. Do not panic. **Allocate your time wisely.**
- Don't forget to submit your fist online self-evaluation form by the end of today.

Name____ ID __ __ __

1. (7 pt) No explanation is needed for this problem.
The *Fourier transform* $M(f)$ of a signal $m(t)$ is shown in Figure 1.
a. (1 pt) Is $m(t)$ a real-valued signal?

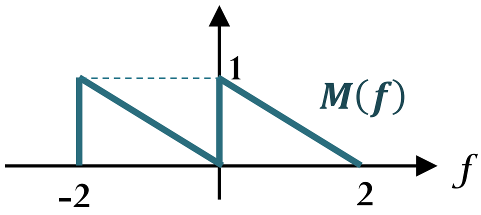
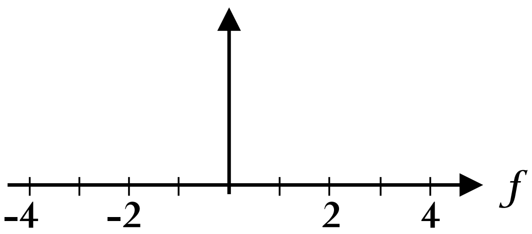
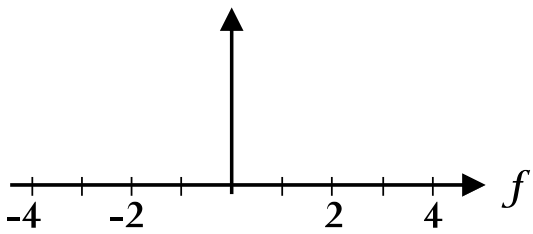


Figure 1

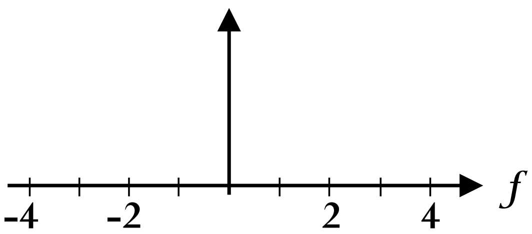
b. (2 pt) Let $x(t) = m(t) \times 4\cos(4\pi t)$.
Carefully sketch $X(f)$.



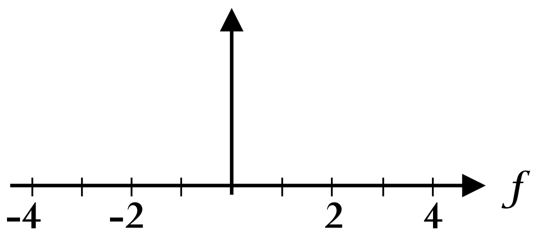
c. (1 pt) Let $g(t) = m(t) \times 2\cos(2\pi t)$.
Carefully sketch $G(f)$.



d. (2 pt) Let $y(t) = m\left(\frac{t}{2}\right)$.
Carefully sketch $|Y(f)|$.



e. (1 pt) Let $z(t) = m\left(\frac{2-t}{2}\right)$. Carefully sketch $|Z(f)|$.

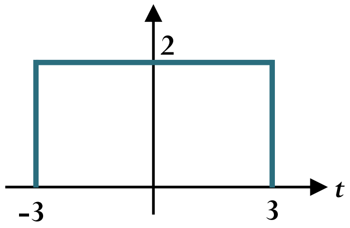
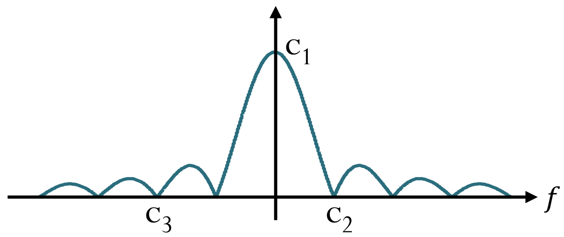
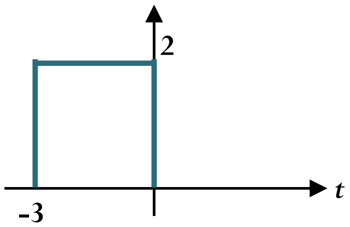
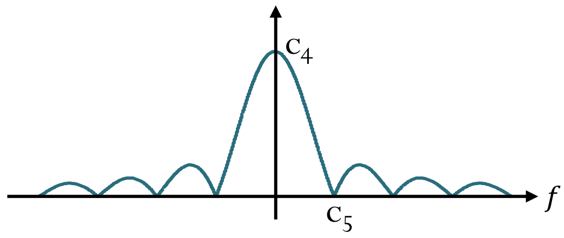
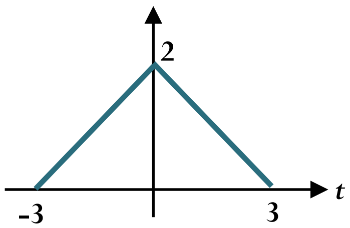
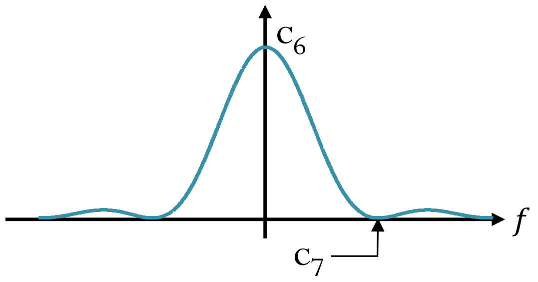


2. (3 pt) Consider the system described in each part below. Is it "linear time-invariant" (LTI)? Indicate with a Y(es) or an N(o) in the rightmost column. Explanation is not needed.

	System	Is LTI?
a.	A system that outputs $y(t) = 3x(t) + 1$ when its input is $x(t)$.	
b.	A channel that delays its input by 1 second.	
c.	A modulator box that multiplys its input by $\sqrt{2}\cos(2000\pi t)$.	

Name____ ID __ __ __

3. (6 + 1* pt) Each part below shows the plot of a signal and the corresponding magnitude plot of its Fourier transform. Find the values of the constants (corresponding to the zeroes and the peaks) shown in the plots. Put your answers in the spaces provided at the end of the question. Explanation is not required.

	$x(t)$	$\lvert X(f)\rvert$
(a)		
(b)		
(c)		

c_1 = ______, c_2 = ______, c_3 = ______, c_4 = ______, c_5 = ______, c_6 = ______, c_7 = ______.

4. (1+1+1* pt) Consider a cosine pulse

$$p(t) = \begin{cases} A\cos(2\pi f_0 t), & t_0 \le t \le t_0 + \Delta, \\ 0, & \text{otherwise.} \end{cases}$$

The magnitude of its Fourier transform is plotted in Figure 2.

Find the following parameters of the pulse from the plot:

a. f_0 = ____________

b. Δ = ____________

c. A = ____________

Note that they are all integers.

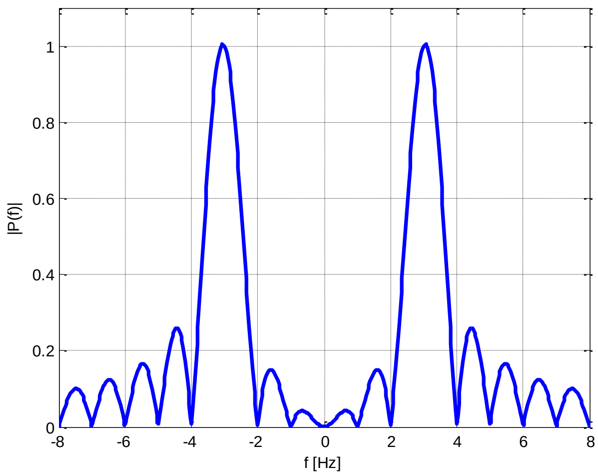


Figure 2

Name____ ID __ __ __

5. (5 pt) Consider the DSB-SC modem with no channel impairment shown in Figure 3. Suppose that the message is band-limited to B = 500 Hz and that f_c = 2,000 Hz.

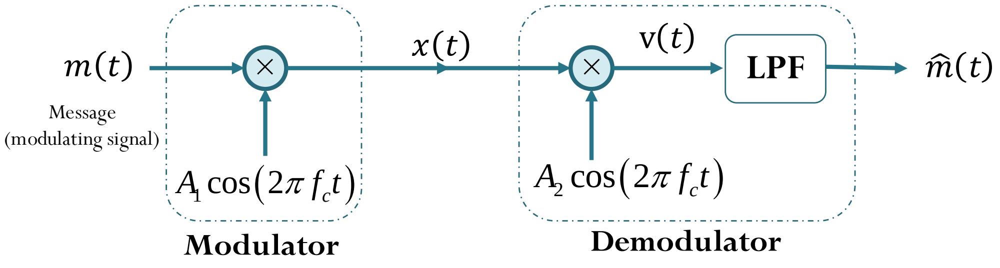


Figure 3

Let $A_1 = 2$ and $A_2 = 3$.

a. (3 pt) Suppose the frequency response of the LPF is

$$H_{LP}(f) = \begin{cases} g, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of g that makes $\hat{m}(t) = m(t)$.

b. (2 pt) Suppose the impulse response of the LPF is $h_{LP}(t) = \alpha \operatorname{sinc}(2016\pi t)$.

Find the value of α so that $\hat{m}(t) = m(t)$. Explanation is not required for this part.

6. (5 pt) Signal $x(t) = 5\cos(2\pi \times 5 \times 10^6 \times t)$ is transmitted. The received signal is $y(t) = 4\cos(2\pi \times 5 \times 10^6 \times t - \phi)$. Assume the phase shift is caused by propagation delay.

a. (3 pt) Suppose $\phi = \frac{\pi}{4}$ [rad]. What is the minimum distance between the transmitter and the receiver?

b. (2* pt) Suppose $\phi = -\frac{\pi}{4}$ [rad]. What is the minimum distance between the transmitter and the receiver?

7. (30 pt) Explanation is not required for this problem.
Suppose a baseband signal $m(t)$ is transmitted via the DSB-SC system shown in Figure 4. The carrier frequency is set at f_c Hz. Furthermore, assume that the **low-pass filter** is ideal with frequency response

$$H_{LP}(f) = \begin{cases} 1, & |f| \le 600, \\ 0, & \text{otherwise.} \end{cases}$$

$m(t)\sqrt{2}\cos\sqrt{2}\cos$
$= m(t) \times 2\cos^2$
$= m(t)(1+\cos(2\pi(2f_c)t))$
$= m(t)$
$+ m(t)\cos(2\pi(2f_c)t)$

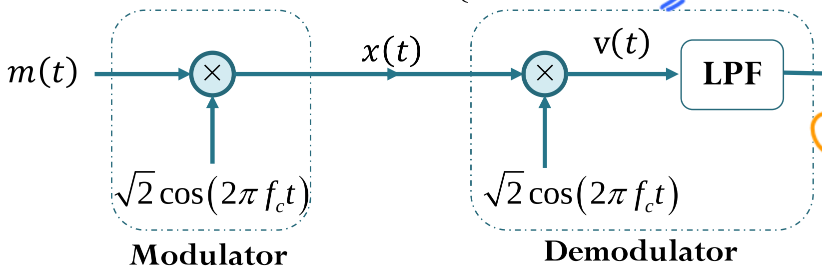
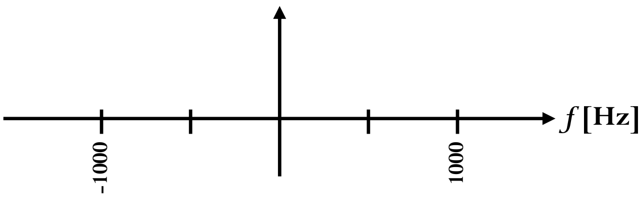


Figure 4

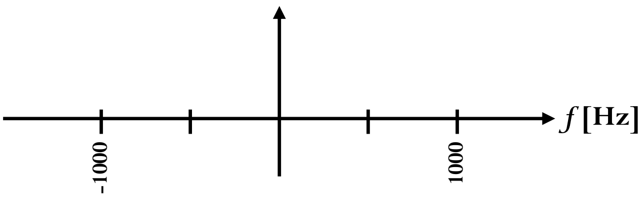
$V(f) = M(f) + \frac{1}{2}M(f-2f_c) + \frac{1}{2}M(f+2f_c)$

a. (3+4+4+2 pt) Suppose $m(t) = \cos 500\pi t$ and $f_c = 1000$ Hz.
Sketch the following signals.

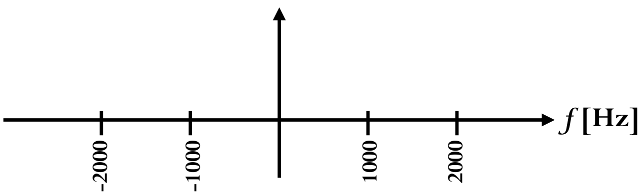
i. The spectrum $M(f)$ of $m(t)$.



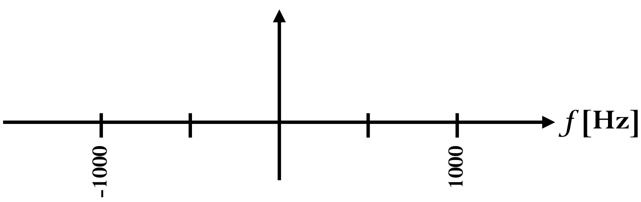
ii. The spectrum $X(f)$ of $x(t)$.



iii. The spectrum $V(f)$ of $v(t)$.



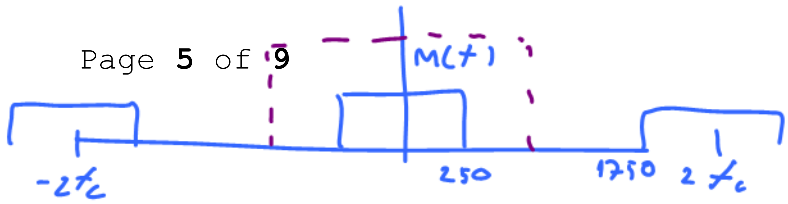
iv. The spectrum $\hat{M}(f)$ of $\hat{m}(t)$.



b. (2+2+2+2+1+1* pt.) Assume $f_c = 1000$ Hz.
For each of the following $m(t)$, find the corresponding $\hat{m}(t)$.

$m(t)$	$\hat{m}(t)$
$m(t) = \cos(500\pi t)$	$\cos(500\pi t)$
$m(t) = \cos(1400\pi t)$	
$m(t) = \cos(5000\pi t)$	
$m(t) = \cos(2600\pi t)$	
$m(t) = \text{sinc}(500\pi t)$	$\text{sinc}(500\pi t)$
$m(t) = \text{sinc}(1400\pi t)$	

Reminder: Although the back of each page will not be graded, it can be used for calculations of problems that do not require explanation (like this one).

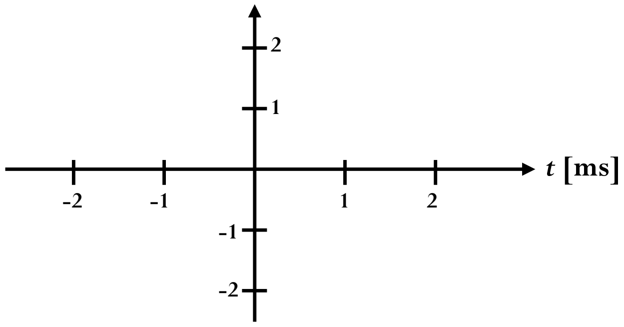


Name____ ID __ __ __

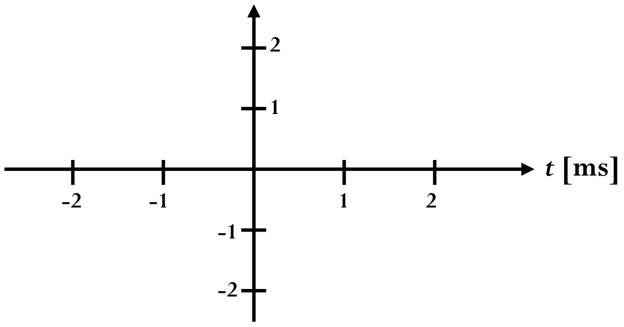
c. (6 pt) Suppose $m(t) = \cos 500\pi t$ and $f_c = 4000$ Hz. milli-

Sketch the following signals for time t between -2 and 2 **m**s.

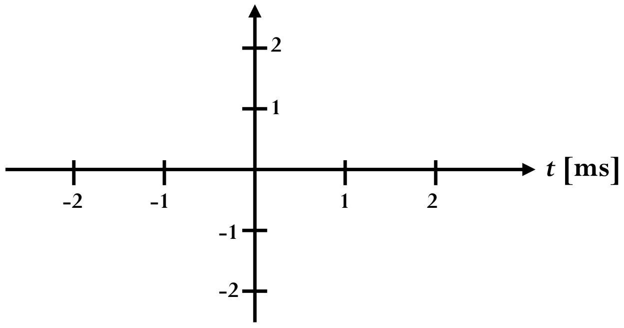
i. The message $m(t)$



ii. The transmitted signal $x(t)$



iii. The signal $v(t)$



d. (1 pt) Assume $f_c = 1000$ Hz. Suppose $m(t) = 1[|t| \le 2]$. Will $\hat{m}(t) = m(t)$?

~~3.~~ (3 pt) Consider the signal $x(t) = 7\cos(40\pi t + 10\sin(4\pi t))$. Find its instantaneous frequency at time $t = 0$.

Name____ ID __ __ __

9. (1×6 = 6 pt) Suppose we want to transmit a baseband message $m(t)$ via modulators of the form shown in Figure 5.

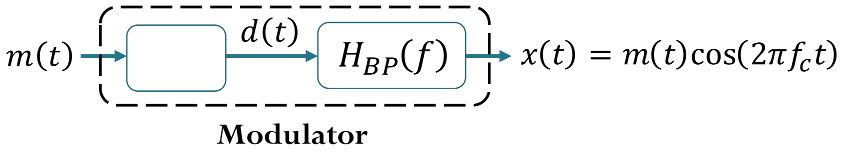


Figure 5

Assume that $m(t)$ is band-limited to B = 10 Hz and that the frequency response of the bandpass filter is

$$H_{BP}(f) = \begin{cases} g, & |f - f_c| \le B, \\ g, & |f + f_c| \le B, \\ 0, & \text{otherwise.} \end{cases}$$

Consider three different modulators in the three parts below. They are different in the way that the signal $d(t)$ in Figure 5 is produced from the message $m(t)$. For each modulator, choose the appropriate value of its carrier frequency f_c and its gain g of the bandpass filter so that $x(t) = m(t)\cos(2\pi f_c t)$.

Modulator	$d(t)$	g	f_c
a.	$d(t) = \left(m(t) + \cos(90\pi t)\right)^2$		
b.	$d(t) = m(t)\cos^2(90\pi t)$		
c.	$d(t) = \left(m(t) + \cos(90\pi t)\right)^3$		

10. (1×4 = 4 pt) Consider a "square" wave (a train of rectangular pulses) $r(t)$ shown in Figure 6. Its value periodically alternates between A and 0 with period T_0. Suppose A = 2.

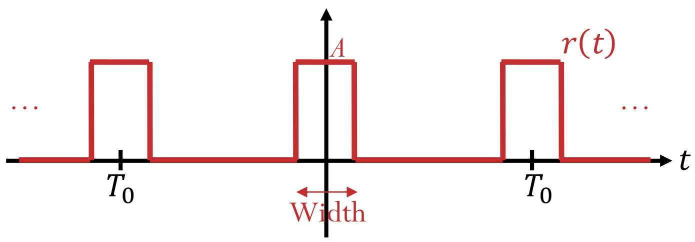


Figure 6

The Fourier series expansion of $r(t)$ is given by $\sum_{k=-\infty}^{\infty} c_k e^{j2\pi(kf_0)t}$ where $f_0 = 1/T_0$.

a. Suppose the duty cycle is 50%. Find c_0 and c_2.

c_0 = ______, c_2 = ______

b. Suppose the duty cycle is 20%. Find c_0 and c_5.

c_0 = ______, c_5 = ______

Name____ ID __ __ __

11. (14 pt) For each of the following signal $g(t)$, find its (normalized) average power $P_g \equiv \left\langle |g(t)|^2 \right\rangle$. No explanation is required here. Put your answers directly in the table. Do not use any approximation.

$g(t)$	$P_g \equiv \left\langle \lvert g(t)\rvert^2 \right\rangle$
(2 pt) $g(t) = 10\cos(10t + 10^\circ)$	
(2 pt) $g(t) = 5\cos(10t + 10^\circ) + 12\cos(10t + 100^\circ)$	
(2 pt) $g(t) = 5\cos(10t + 10^\circ) + 12\cos(100t + 100^\circ)$	
(2 pt) $g(t) = \left(10\cos(10t + 10^\circ)\right)^2$	
(2 pt) $g(t) = \left(5\cos(10t)\right)^2 + 12\cos(20t)$	
(1 pt) $g(t) = 1\left[\lvert t\rvert \le 10\right]$	
(1 pt) $g(t) = \sum_{k=0}^{35} \cos\left(10t + (k \times 10^\circ)\right)$	
(1 pt) $g(t) = \sum_{k=0}^{36} \cos\left(10t + (k \times 10^\circ)\right)$	
(1* pt) $g(t) = \text{LPF}\left\{\left(2\cos(10t + 10^\circ)\right)^3\right\}$ where the frequency response of the LPF is given by $H(f) = \begin{cases} 1, & \lvert f\rvert > 3, \\ 0, & \text{otherwise.} \end{cases}$	

12. (4 pt) Calculate the energy of the signal $g(t)$ shown in Figure 7.

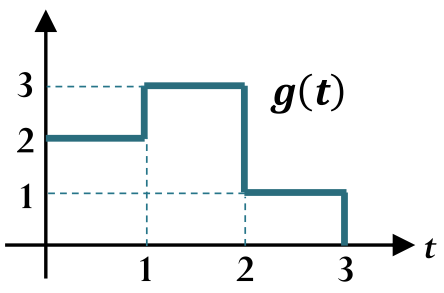


Figure 7

Name____ ID _ _ _

13. (5 pt) Consider an LTI communication channel. Suppose when we put

$$x(t) = 2\cos(2\pi t) + 4\cos(4\pi t) + 6\cos(6\pi t) + 7\cos(8\pi t) + 1$$

into this channel, we get

$$y(t) = \cos(2\pi t) + 4\cos(4\pi t) + 3\sin(6\pi t) + 0.5$$

as its output. Explanation is not required for this question.

a. (4 pt) Let $|H(f)|$ be the amplitude (magnitude) responses of the channel that satisfies the above input-output relation.

i. (2 pt) Find $|H(2)|$.

ii. (2 pt) Find $|H(4)|$.

b. (1 pt) Is this channel distortionless?

14. (3 pt) Consider the impulse train $g(t)$ shown on the left in Figure 8. Plot its Fourier transform $G(f)$ from $f = -2$ to $f = 2$. Explanation is not required for this question.

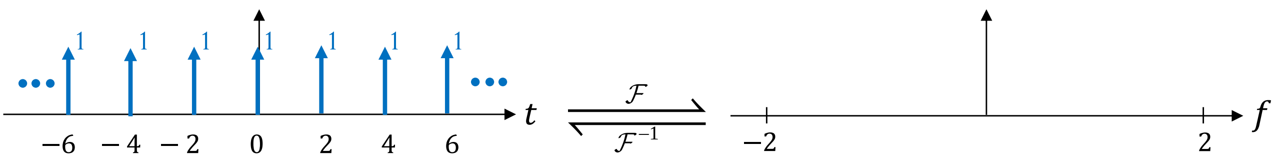


Figure 8

15. (1 pt)

a. (1 pt) Do not forget to submit your study sheet with your exam.

b. Make sure that you write your name and ID on every page. (Read the instruction on the cover page.)

c. The online self-evaluation form is due by the end of today.

$$2\cos^2 x = 1 + \cos(2x) \qquad 2\sin x \cos x = \sin(2x)$$

$$2\sin^2 x = 1 - \cos(2x)$$

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t}\, dt$$

$$\cos(2\pi f_c t + \theta) \xrightleftharpoons{\mathcal{F}} \frac{1}{2}\delta(f - f_c)e^{j\theta} + \frac{1}{2}\delta(f + f_c)e^{-j\theta}$$

$$g(t - t_0) \xrightleftharpoons{\mathcal{F}} e^{-j2\pi f t_0} G(f)$$

$$e^{j2\pi f_0 t} g(t) \xrightleftharpoons{\mathcal{F}} G(f - f_0)$$

$$g(t)\cos(2\pi f_c t) \xrightleftharpoons{\mathcal{F}} \frac{1}{2}G(f - f_c) + \frac{1}{2}G(f + f_c)$$

$$1[|t| \le a] \xrightleftharpoons{\mathcal{F}} 2a\,\mathrm{sinc}(2\pi f a) \text{ where } \mathrm{sinc}(x) = \frac{\sin x}{x}$$